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**RANGE AND TRACKING ACCURACY  
OF AN/APG-15B**

**REPORT**

**875**

**RADIATION LABORATORY  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
CAMBRIDGE - MASSACHUSETTS**

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Report 875

March 22, 1946

RANGE AND TRACKING ACCURACY OF AN/APG-15B

Abstract

Flights to test the range and tracking accuracy of AN/APG-15B were made in a TB-24 bombing plane with various planes as targets. The film record of readings of radar range dials and oscilloscopes was compared with the photographs of the target planes taken from the tail of the bomber.

Analysis shows that AN/APG-15B measures air-to-air range as well as photographic sighting does, the Old Range Unit measuring the range slightly less accurately than the New Range Unit. It was established that the tracking or gun pointing accuracy is unaffected by the computer, results with computer in and computer out being indistinguishable. The time lag between the turret sight and the indicator spot was on the average .31 seconds, when the time constant of the system was approximately one-fourth of a second; with a smaller time constant for the system, approximately one tenth of a second, the lag was on the average .13 seconds. The turret was slewed in elevation  $+2^\circ$ ,  $+4^\circ$  and in azimuth  $+2^\circ$ ,  $+4^\circ$ ,  $+30^\circ$ . The attempt to establish a functional relation between scope deflection and target deflection was not entirely successful, but analysis does indicate an approximate linear relation.

High altitude tests were not made because of trouble which developed in the engines of the bombing plane above 10,000 feet. Flights over water at 500 feet, with a stationary target such as a freighter, were unsuccessful because they yielded no analyzable film.

C. T. Bumer

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Title page  
20 numbered pages  
13 pages of figures

## **RANGE AND TRACKING ACCURACY OF AN/APG-15B**

### **Introduction: Flight Plan**

In accordance with plans drawn up on May 3, 1945, by Group 91.5, flights were made in a TB-24 bombing plane, with cameras installed to photograph a target plane, radar dials, and oscilloscopes, in an attempt to test the range and tracking accuracy of AN/APG-15B. The plans called for 14 missions, numbered 1 to 14 inclusive.

Missions 1, 2, 3, 4, to test air-to-air accuracy at 10,000 feet, were flown and the resulting film analyzed.

Mission 5, to test air-to-air range accuracy at maximum altitude was attempted, but because of motor trouble in the TB-24 at high altitudes the flights were cancelled.

Missions 6A, 6B, to search for a relationship between deflection of indicator spot on the scope and the target deflection, were flown and the film was analyzed.

Mission 7, to discover the time lag between turret sighting and indicator spot on the scope, was flown and the film was analyzed.

Missions 8 and 9, to test pointing accuracy with the computer in and computer out, were flown and the film was analyzed.

Mission 10, to test air-to-air range accuracy of the Old Range Unit was flown. This mission is similar to Mission 2 and 4. The film was analyzed.

Mission 11, to test spot sensitivity and jitter, was flown, but all film was either jammed or overexposed, and no analyzable record was forthcoming.

Mission 12, to discover time lag between turret sighting and indicator spot on the scope, was flown. This mission differed from Mission 7 in that the time constant of the system was one tenth of a second. The film was analyzed.

Missions 13 and 14, to test range and tracking accuracy on low altitude flights over water were flown, with the target a small freighter ship. No analyzable film was forthcoming.

## I Missions 1, 2, 3, 4 : Air-to-Air Range Accuracy

The air-to-air range accuracy of AN/APG-15B was studied in a series of flights with cameras installed in a TB-24 bombing plane. The target or pursuing plane was an FM-2, and both planes were flying level at 10,000 feet. Runs were made at closing rates of 25, 50, 75 and 100 miles per hour, with the TB-24 flying at 180 miles per hour. Flights were made at Bedford Field from June 15 to September 15, 1945.

To photograph dials and scope, GSAP (Gun Sight Aiming Point) cameras N4A (24 volts), modified to use Wollensak 17 mm. lens, were used. The tail camera was a GSAP camera with 3-inch lens for photographing the target with larger image than is possible with a 35 mm. lens, and with a red filter for emphasizing contrast between the image of the target plane and the background. The larger image and the clearer outlines permit more accurate measurement. Synchronization of the several cameras was accomplished by the use of a coding box, especially designed for GSAP cameras, that employed a constant speed motor and a gear reduction box fitted with a cam that actuated a microswitch. This switch energized a relay which controlled the over-run indicators in the cameras. The installation of these several improvements did much to improve the results of the analysis by permitting more accurate measurements of the image and by making synchronization easier to obtain and better.

Photographic range data were obtained by projecting the pictures taken by the nose camera and measuring the distance between two fiduciary marks on the FM-2 target plane. These marks were two white lines painted on the wings of the plane and the distance was measured from center to center. The distance from the center of one wing stripe to the center of the other on the actual airplane was taken as the mean of eight measurements made by four different persons, accurate to four significant figures. For the FM-2, this distance was 28.27 feet. Camera range was then calculated from the formula  $R = \frac{K T}{w}$ , where R is range in yards, T is 28.27 feet, w is the length of the image in inches (measured to 1/64 of an inch), and K is a constant which (in theory) depends on the focal length of the camera lens, the focal length of the projector lens, and the distance between the projector and the screen. Actually, K was found by measuring carefully the projections of ground objects of known size at known distances. The calibration data were graphed on a large scale, and photographic range was interpolated from the calibration curve.

Radar range was read directly from the pictures of the range dial.

Errors due to measuring, focussing and calibration for a target 25 feet long were interpolated from graphs in "Errors in Optical Range Determination" by P. R. Halmos, Report #91.5.

Following is a table of errors.

<u>Range in yards</u>	<u>Error in yards</u>	<u>% Error</u>
1000	15	2.50
950	23	2.42
900	21	2.33
850	19	2.24
800	17	2.13
750	16	2.13
700	14	2.00
650	12	1.85
600	10	1.67
550	8	1.45
500	7	1.40

The results of Missions 1, 2, 3, 4 are as follows:  
Mission 1, closing rate 25 miles per hour.

<u>Film Numbers</u>	<u>Runs</u>	<u>Comparable Range in Yards</u>	<u>Error</u>
F-268, F-306	1	870 - 260 = 610	From 870 to 662, error 4 A.E.*
	2	923 - 260 = 663	" 923 to 850, " 2 "
	3	970 - 260 = 710	" 970 to 692, " 2 "
F-241, F-219	2	997 - 260 = 737	" 997 to 361, " 2 "
	1	1010 - 260 = 750	" 1010 to 542, " 2 "
F-243, F-205	4	983 - 260 = 723	" 983 to 258, " 2 "
F-194, F-216	1	1010 - 260 = 750	" 1010 to 325, " 2 "
	2	1024 - 260 = 764	" 1024 to 333, " 2 "

The statement, "Differences are less than allowable errors", means that, except for one or at the most two readings, the differences are less than the errors allowed for measuring, focussing, and calibrating, according to Halmos' "Error Study".

For closing rate of 25 miles per hour, radar range and photographic range can be compared from 973 yards to 311 yards, a distance of 662 yards. On the average, from a distance of 973 yards to 503 yards, a total of 470 yards, the differences between radar range and photographic range are less than the allowable errors. Percentage error allowed decreases from 2.5 at 1000 yards to 1.4 at 500 yards. The systematic error, i.e., the mean of the differences between radar range and photographic range, is 17.9 yards.

Mission 2, closing rate 50 miles per hour.

<u>Film Numbers</u>	<u>Runs</u>	<u>Comparable Range in Yards</u>	<u>Error</u>
F-320, F-197	1	959 - 260 = 699	From 959 to 530, error 4 A.E.
	2	945 - 260 = 685	" 945 to 764, " 2 "
	3	945 - 260 = 685	" 945 to 655, " 2 "
F-314, F-243	1	958 - 230 = 698	" 958 to 515, " 2 "
	2	946 - 260 = 685	" 946 to 580, " 2 "
	3	901 - 260 = 641	" 901 to 579, " 2 "
F-204, F-250	1	935 - 260 = 675	" 935 to 331, " 2 "
	2	923 - 260 = 663	" 923 to 343, " 2 "
	3	983 - 260 = 723	" 983 to 331, " 2 "

\* Allowable error.

875-3

For closing rates of 50 miles per hour, radar range and photographic range can be compared from 944 yards to 260 yards, a distance of 684 yards. On an average, from 944 yards to 514 yards, a total of 430 yards. Differences are less than the allowable error. Percentage error allowed decreases from 2.4 at 950 yards to 1.4 at 500 yards. The systematic error is 8.7 yards.

Mission 3, closing rate 75 miles per hour.

<u>Film Numbers</u>	<u>Runs</u>	<u>Comparable Range in Yards</u>	<u>Error</u>
F-261, F-307	1	900 - 260 = 640	From 900 to 467, error < A.E.
	2	881 - 260 = 621	" 841 to 374, " < "
	3	841 - 260 = 581	All errors
	4	870 - 260 = 610	" 870 to 500, " < "
F-316, F-209	1	902 - 260 = 642	" 902 to 520, " < "
	2	869 - 260 = 609	" 869 to 485, " < "
	3	841 - 260 = 581	" 841 to 598, " < "
	4	859 - 260 = 599	" 859 to 525, " < "
F-220, F-254	1	902 - 260 = 642	" 902 to 386, " < "
	2	902 - 260 = 642	" 902 to 343, " < "
F-223, F-249	1	923 - 260 = 663	" 840 to 363, " < "
	2	934 - 260 = 674	" 934 to 368, " < "
	3	912 - 260 = 652	" 912 to 379, " < "
	4	890 - 332 = 558	" 890 to 366, " < "

For closing rate of 75 miles per hour, radar range and photographic range can be compared from 879 yards to 265 yards, a distance of 614 yards. On an average, from 879 yards to 424 yards, a total distance of 455 yards, differences are less than the allowable errors. Percentage error allowed decreases from 2.3 at 900 yards to 1.4 at 500 yards. The systematic error is 6.7 yards.

Mission 4, closing rate 100 miles per hour.

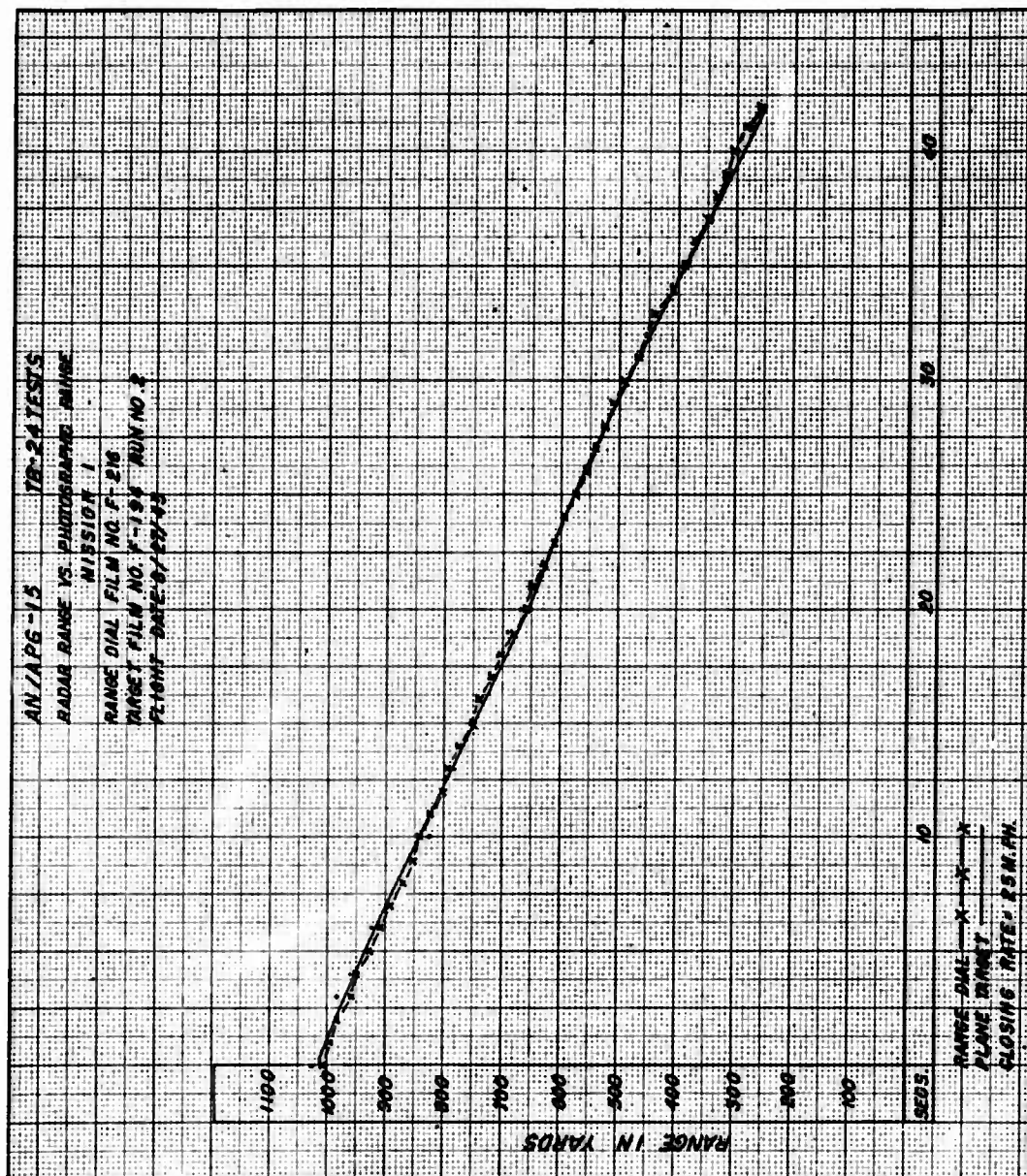
<u>Film Numbers</u>	<u>Runs</u>	<u>Comparable Range in Yards</u>	<u>Error</u>
F-243, F-205	1	850 - 260 = 590	From 850 to 336, error < A.E.
	2	880 - 260 = 620	" 880 to 330, " < "
	3	764 - 260 = 504	" 764 to 343, " < "
F-220, F-254	3	865 - 260 = 605	" 865 to 314, " < "
	4	850 - 260 = 590	" 850 to 402, " < "

For closing rate of 100 miles per hour, radar range and photographic range can be compared from 842 yards to 260 yards, a distance of 582 yards. On an average, from 842 yards to 355 yards, a total distance of 487 yards, differences are less than the allowable errors. Percentage error allowed decreases from 2.2 at 850 yards to 1.4 at 500 yards. The systematic error is 3.6 yards.

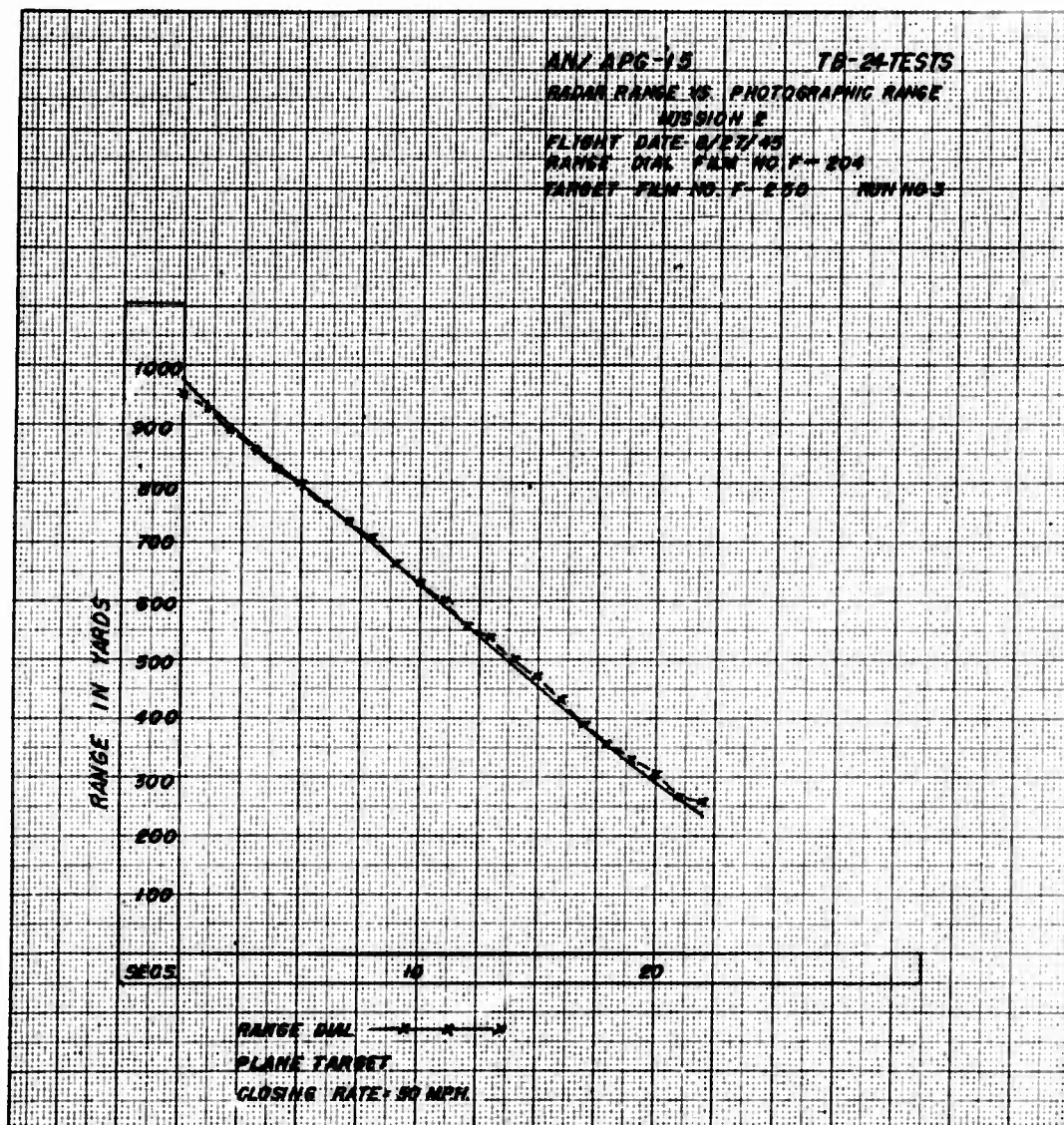
Thirty-six runs were made and the resulting film was analyzed. In general, the tail camera photographed the target plane at 950 yards sufficiently distinctly to provide a measurable image, and photographed it well up to 200 yards; the radar dial registered

range from 1000 yards up to 260 yards, where invariably the dial stuck. The range over which the two measurements could be compared was about 640 yards. Over a range of approximately 450 yards of this distance, the differences between the range readings of the radar dial and calculations of range from photographs of the target plane are less than the allowable error in measuring, focussing, and calibration. The duration of the runs was of course inversely proportional to the closing speed, from 20 to 30 seconds for the 25 miles per hour speed, and from 5 to 9 seconds for the 100 miles per hour speed. The results indicate that AN/APG-15B measures air-to-air range as well as it can be determined by photographic methods. As the closing rate increases, the systematic error decreases. For photographic range of 400 yards and less, the differences between radar range and photographic range are slightly greater than the allowable errors.

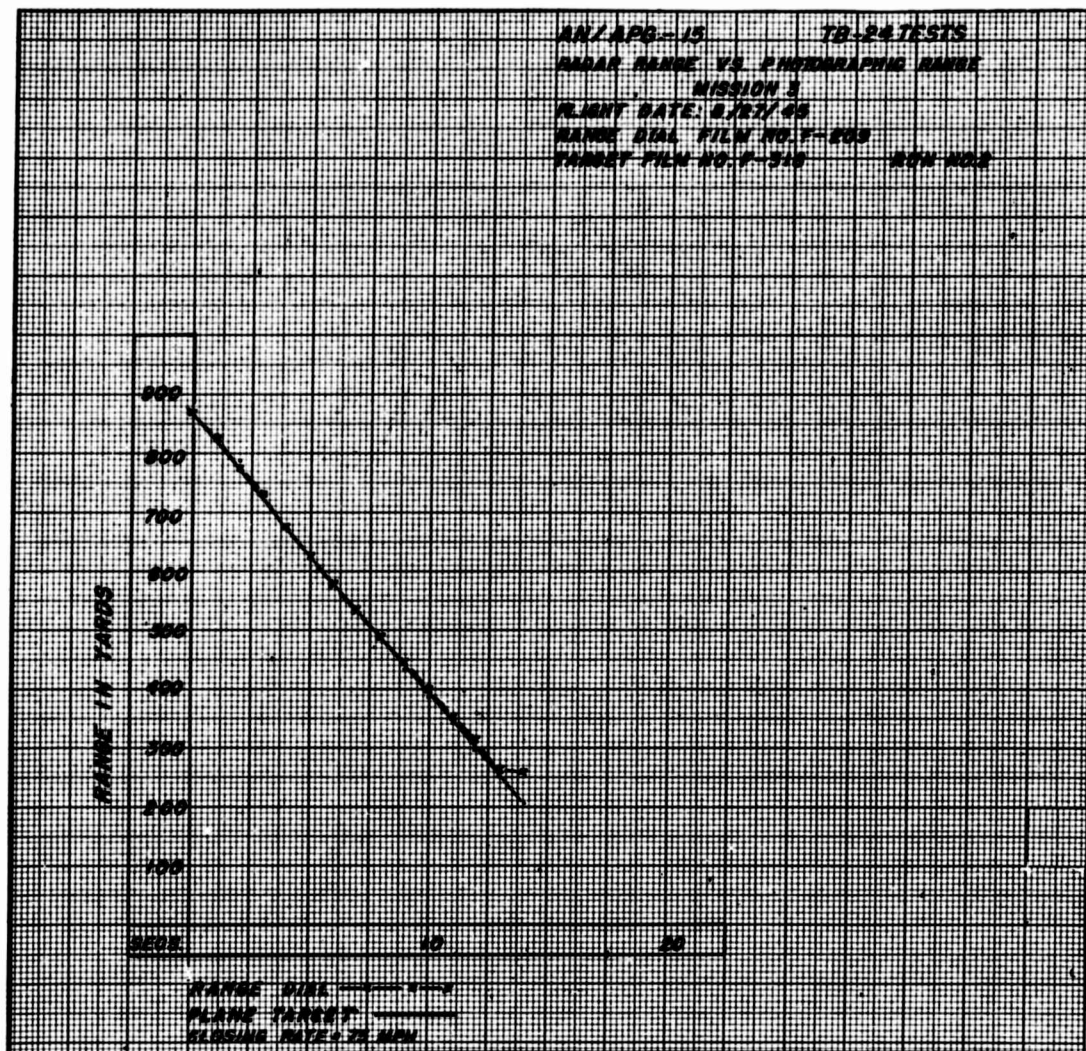
Graphs showing radar range and photographic range for each closing rate are included in this report.



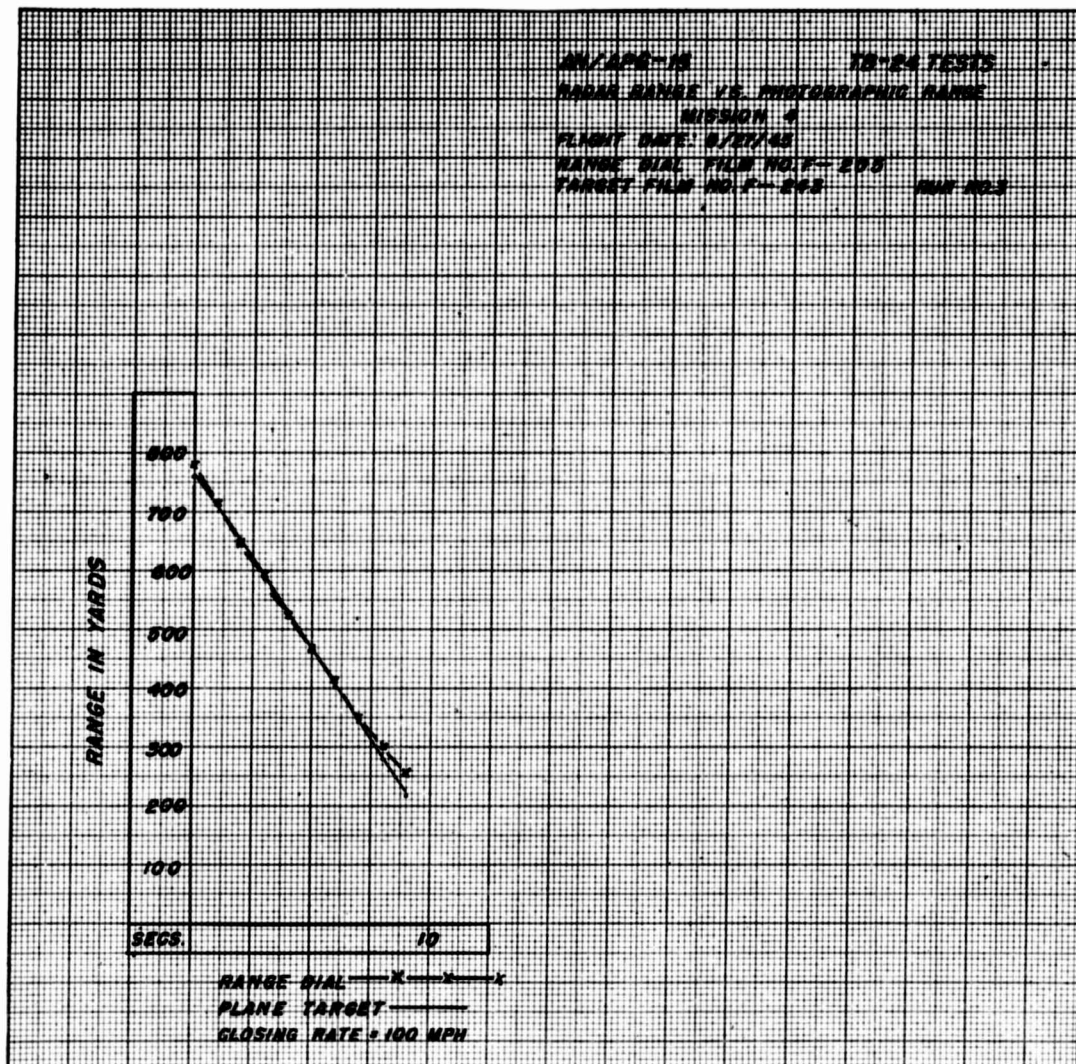
X-7291-A



X-7281-A



X-7272-B



X-7276-B

## II Mission 10 : Air-to-Air Range Accuracy, Old Range Unit

Flights were made to check air-to-air range accuracy of the Old Range Unit and the results were compared with those of Missions 2 and 4, the air-to-air range tests of the New Range Unit. By Old Range Unit is meant the unit referred to in Radiation Laboratory Report M-215, Appendix A, with modifications a and b; New Range Unit is model CP-8C.

Closing rate 50 miles per hour.

F-432, F-425	Run 1	892-260 = 632	892 to 558, error $\angle$ allowable error
	2	901-260 = 641	901 to 597, " $\angle$ "
	3	924-260 = 644	924 to 726, " $\angle$ "
	4	924-260 = 664	924 to 534, " $\angle$ "
F-415, F-423	1	923-260 = 663	923 to 726, " $\angle$ "
	2	901-260 = 641	901 to 662, " $\angle$ "
	3	912-260 = 652	912 to 541, " $\angle$ "
	4	901-260 = 641	901 to 449, " $\angle$ "

From 909 yards to 294 yards, a distance of 615 yards, on the average, the two ranges were comparable. Differences between the ranges were less than the allowable error for only a short distance, from 909 yards to 621 yards, a distance of 288 yards. The systematic error is 12.3 yards.

Closing rate 100 miles per hour.

F-414, F-412	Run 1	840-160 = 580	840 to 550, error $\angle$ allowable error
	2	880-260 = 620	none
	3	996-260 = 736	none
	4	912-260 = 652	none
	5	912-260 = 652	very few
F-438, F-190	1	956-260 = 686	946 to 668, " $\angle$ "
	2	880-260 = 620	880 to 833, " $\angle$ "
F-438, F-190	3	859-260 = 599	859 to 679, " $\angle$ "
	4	850-260 = 590	850 to 444, " $\angle$ "
	5	322-260 = 562	very few
	6	890-332 = 558	890 to 332 " $\angle$ "

From 890 yards to 267 yards, a distance of 623 yards, the radar range and the photographic range could be compared. Differences between these were less than the allowable errors for only 160 yards, from 890 yards to 730 yards. The systematic error is 12.2 yards.

Errors due to measuring, focussing, and calibrating are a function of range. To compare the results of Mission 10 with those of Missions 2 and 4, it is necessary to compare differences at approximately the same range. The table immediately following gives differences for both Mission 10 and 2 at 850 yards, 800 yards, 700 yards, 600 yards, and 500 yards.

Closing rate, 50 miles per hour.

	Mission 2	Mission 10	
Range in yards	Mean error in yards	Mean error in yards	Allowable error in yards
850	8.3	13.1	19
800	9.2	13.6	17
700	10.2	12.8	14
600	7.3	11.9	10
500	10.5	14.1	7

Closing rate, 100 miles per hour.

	Mission 4	Mission 10	
Range in yards	Mean error in yards	Mean error in yards	Allowable error in yards
850	9.5	14.1	19
800	13.8	19.7	17
700	10.2	11.8	14
600	9.4	18.2	10
500	5.4	14.6	7

According to the normal curve, only the differences at 600 yards and 500 yards, closing rate 100 miles per hour, are definitely significant. However, the mean errors for the Old Range Unit (Mission 10) are always greater than the mean errors for the New Range Unit. Tests and subsequent analysis for the two units were carried out under conditions as nearly alike as possible. Both units measure range well, but the New Unit measures range more accurately than the old unit.

If we assume that the photographic range data are wrong by the largest amounts possible because of measuring, focussing, and calibrating, and add these amounts to the differences between radar range and photographic range, the resulting errors are in general not greater than 30 yards, particularly for the New Range Unit. The AGS (Airborne Gun Sight) system has been considered successful if the error is not greater than 30 yards.

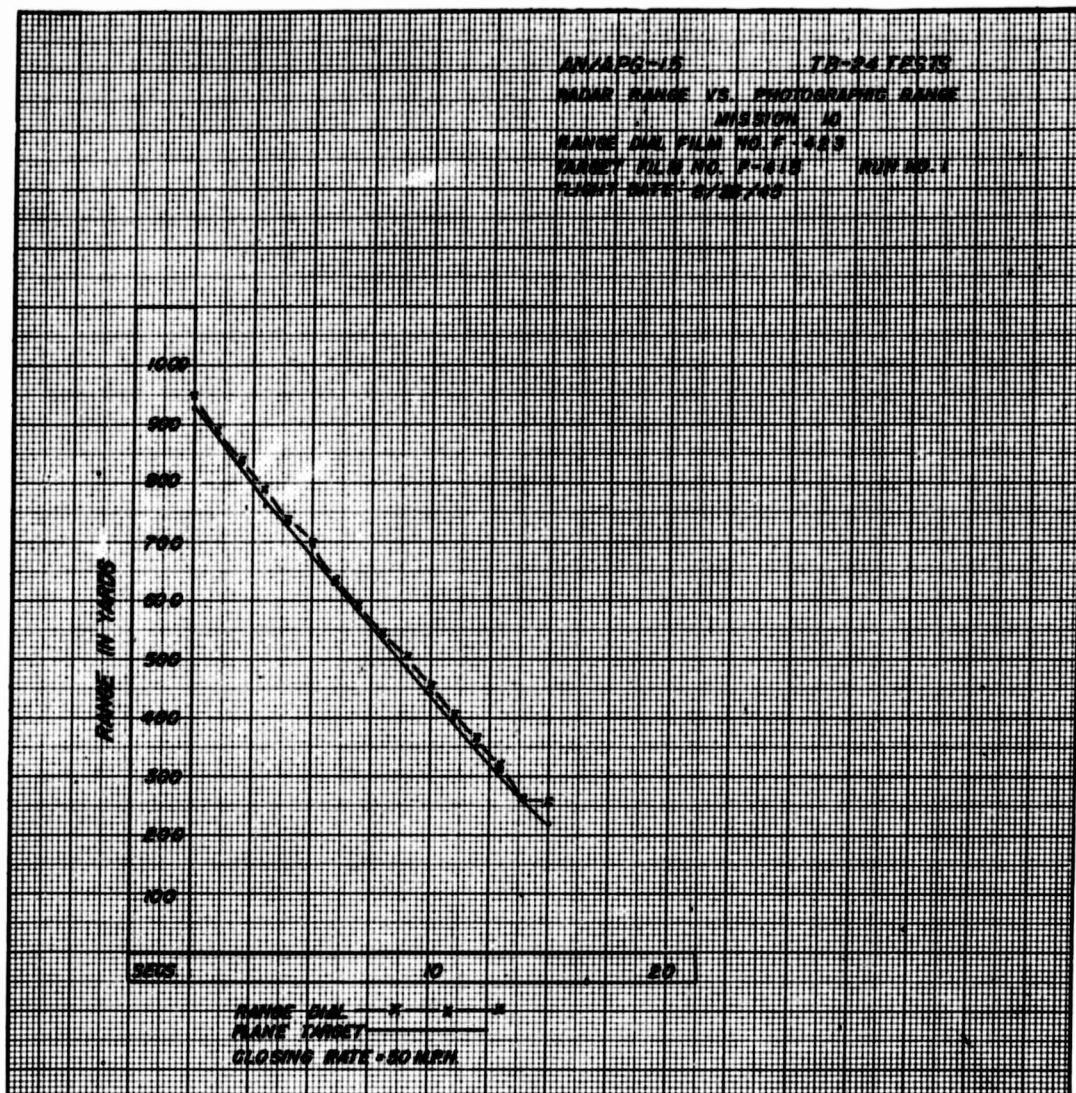
Closing rate, 50 miles per hour.

Range in yards	Mean error plus allowable error.	Mean error plus allowable error.
850	27.3	32.1
800	26.2	30.6
700	24.2	26.8
600	17.3	21.9
500	17.5	21.1

Closing rate, 100 miles per hour.		
Range in yards	Mission 4 Mean error plus allowable error.	Mission 10 Mean error plus allowable error.
850	26.5	33.1
800	30.8	36.7
700	24.2	25.8
600	19.4	26.2
500	12.4	21.6

In Missions 1, 2, 3, 4, and 10 adjustments in the range because of the time lag in the radar system would tend to bring the radar range closer to the photographic range.

Graphs showing radar range and photographic range for both closing rates are included in this target.



X-7300-A

### III Mission 7 : Time lag between Indicator Spot and Turret Sighting

In a test designed to measure the time lag between the turret sight and the indicator spot of the scope, cameras were installed in the TB-24 bombing plane and the target plane used was an AT-11. The tail camera was mounted on the gun. The turret was slewed in azimuth through  $\pm 20^\circ$ ,  $\pm 40^\circ$ ,  $\pm 30^\circ$  and in elevation through  $\pm 20^\circ$ ,  $\pm 40^\circ$  at three rates of speed: slow, medium, and fast. Medium rate of slewing corresponds approximately to the average aim-wander period. The time constant of the radar system was approximately  $1/4$  of a second. (See Appendix)

Several runs were not analyzable because of bad coding. The results of the analysis were graphed with time on the horizontal axis, amplitude on the vertical. The time lag was found by measuring the horizontal distance between the points on the target curve and the scope curve in the same phase where they crossed the time-axis, and by the horizontal distance between the maximum points and minimum points, and averaging the readings.

<u>Film No.</u>	<u>Run</u>	<u>Tracking</u>	<u>Average Time Lag</u>
T: SN-5	2	slow, $\pm 20^\circ$ elevation	.26 seconds
S: SN-6	3	medium $\pm 20^\circ$ elevation	.31 seconds
	4	fast $\pm 20^\circ$ elevation	not analyzed
T: SN-1	4	slow $\pm 40^\circ$ elevation	not analyzed
S: SN-2			
T: SN-3	1	medium $\pm 40^\circ$ elevation	.30 seconds
S: SN-4	2	fast $\pm 40^\circ$ elevation	.25 seconds
	3	slow $\pm 20^\circ$ azimuth	.34 seconds
	4	medium $\pm 20^\circ$ azimuth	.35 seconds
T: SN-5	1	fast $\pm 20^\circ$ azimuth	.29 seconds
S: SN-6			
T: SN-1	1	slow $\pm 40^\circ$ azimuth	.43 seconds
S: SN-2	2	medium $\pm 40^\circ$ azimuth	.36 seconds
	3	fast $\pm 40^\circ$ azimuth	not analyzable
T: SN-9	2	slow $\pm 30^\circ$ elevation	.28 seconds
S: SN-10	3	medium $\pm 30^\circ$ elevation	.28 seconds
	1	fast $\pm 30^\circ$ azimuth	not analyzable
T: SN-11	1	fast elevation typically bad tracking	.31 seconds
S: SN-14	2	azimuth phasing off $10^\circ$ in clockwise direction	.31 seconds

Median Time Lag = .31 seconds

Mean Time Lag = .31 seconds

There seems to be no appreciable difference for different rates of tracking, or for different ways of slewing, the total range of time lag being from .25 seconds to .43 seconds. The test indicates that the scope indicator spot lags in time .31 seconds behind the target image.

IV Mission 12 : Time Lag between Indicator Spot and Turret Sighting,  
Different Time Constant

Mission 12 was similar to Mission 7 except that the time constant of the radar system was one tenth of a second approximately.

<u>Film No.</u>	<u>Run</u>	<u>Tracking</u>	<u>Average Time Lag</u>
T: F-176	2	slow + 2° elevation	.08 seconds
S: F-160	3	medium + 2° elevation	.12 seconds
	4	fast + 2° elevation	.18 seconds
	8	slow + 4° elevation	.13 seconds
T: F-177			
S: F-164	1	fast + 4° elevation	.10 seconds
T: F-176			
S: F-160	1	fast + 2° azimuth	.13 seconds
	5	slow + 4° azimuth	.19 seconds
	7	fast + 4° azimuth	.11 seconds
T: F-177	3	typically bad tracking	.16 seconds
		+ 30° azimuth	
S: F-164			

Median Time Lag .13 seconds

Mean Time Lag .13 seconds

Again there seems to be no appreciable difference for different rates or for different slewing. The total range of time lag is from .08 seconds to .19 seconds. The test indicates that the scope indicator spot lags in time .13 seconds behind the target image.





# V Missions 6A, 6B : Linearity of Scope Deflection

The purpose of Missions 6A, 6B was to search for a functional relationship between the deflection of the indicator spot on the radar scope and the deflection of the photographic sighting point, when AN/APG-15B tracks a target airplane at a range of from 800 to 1000 yards, altitude 10,000 feet. The angle of deflection of the target is the angle between the line of gun sight and the line of sight to the target; the deflection of the scope is the angle between the line from the camera to the center and the line to the pip on the scope. (See Figure 1) This angle is measured by the distance from the boresight point to the target, and for the scope by the distance from the center to the pip. These distances were measured for every eighth frame, i.e., for every half-second of the burst, and the mean of these distances was taken as the measure of the deflection. Sighting angles of 2°, 4°, 6° in directions 12 o'clock, 3 o'clock, 6 o'clock, 9 o'clock were used. Boresight points were set in flight.

Missions 6A, 6B differ only in that different indicator amplifiers were used in an effort to offset the possible bias of a single indicator amplifier.

Results seem to show that the scope deflection is approximately a linear function of the target deflection. See Figures 2, 3, 4, 5. In these figures, deflections at 12 o'clock were taken as positive, those at 6 o'clock as negative for each sighting angle; similarly for deflections at 3 o'clock and at 9 o'clock.

Results of the tests follow.

		6A	
Set Angle		Target Deflection (inches)	Scope Deflection (inches)
2°	12 o'clock	.78	2.58
4°		Not analyzable	Not analyzable
6°		2.81	1.83
2°	3 o'clock	.86	1.96
4°		2.51	2.72
6°		2.83	4.06
2°	6 o'clock	1.09	.62
4°		1.98	2.70
6°		3.19	3.69
2°	9 o'clock	Not analyzable	Not analyzable
4°		1.61	3.19
6°		2.98	3.67

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	Set Angle	Target Deflection (inches)	Scope Deflection (inches)
2°	12 o'clock	1.52	1.22
4°		1.09	.84
6°		2.93	2.09
2°	3 o'clock	1.41	1.26
4°		1.95	1.54
6°		2.83	3.04
2°	6 o'clock	1.23	.34
4°		1.85	1.13
6°		3.12	2.08
2°	9 o'clock	1.39	.86
4°		Not analyzable	Not analyzable
6°		2.85	2.70

AN/APG-15B TB-24 TESTS  
DEFLECTION ANGLE  
MISSION 6

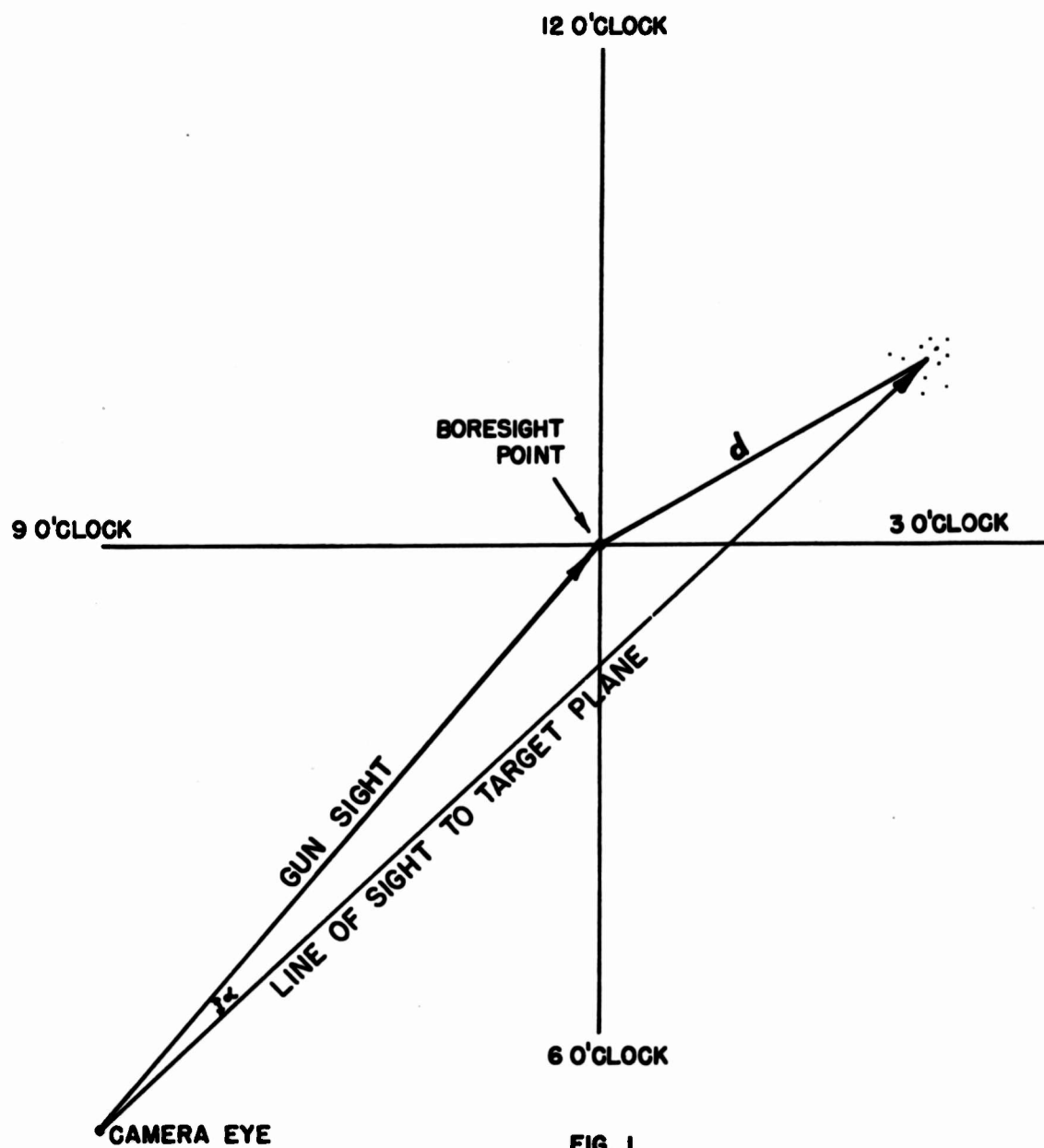


FIG. 1

NOTE:  $\alpha$  IS THE ANGLE OF DEFLECTION

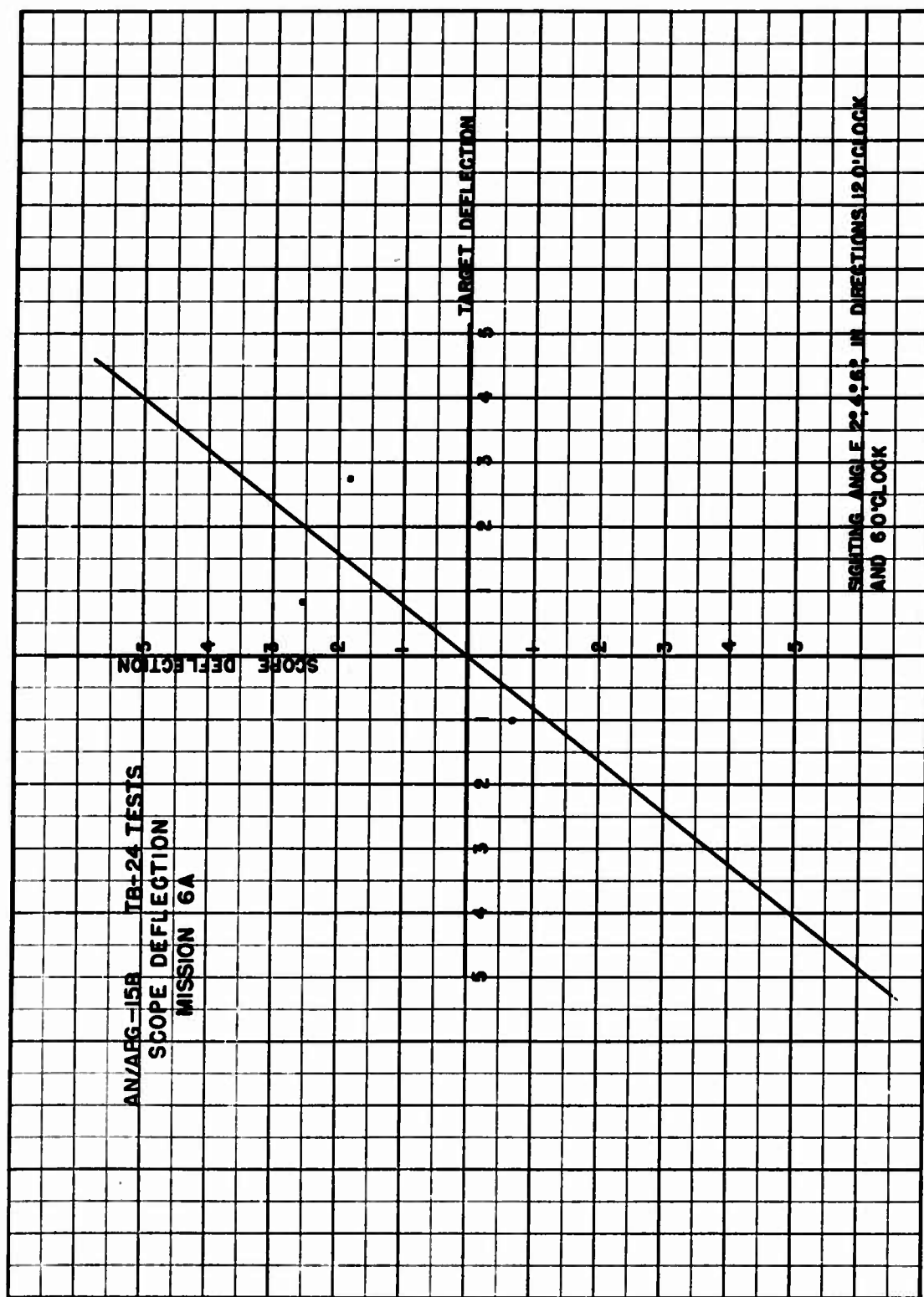
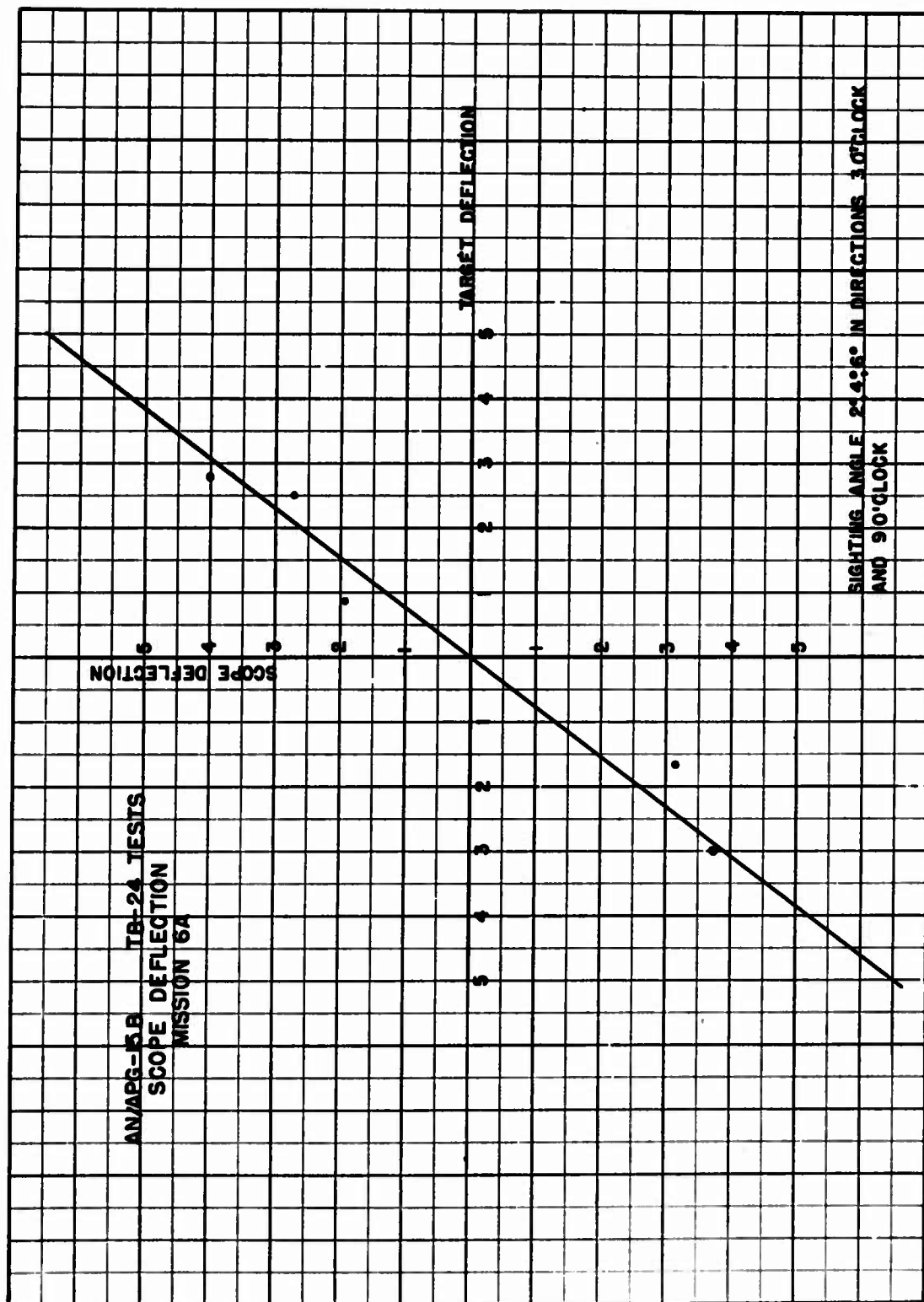
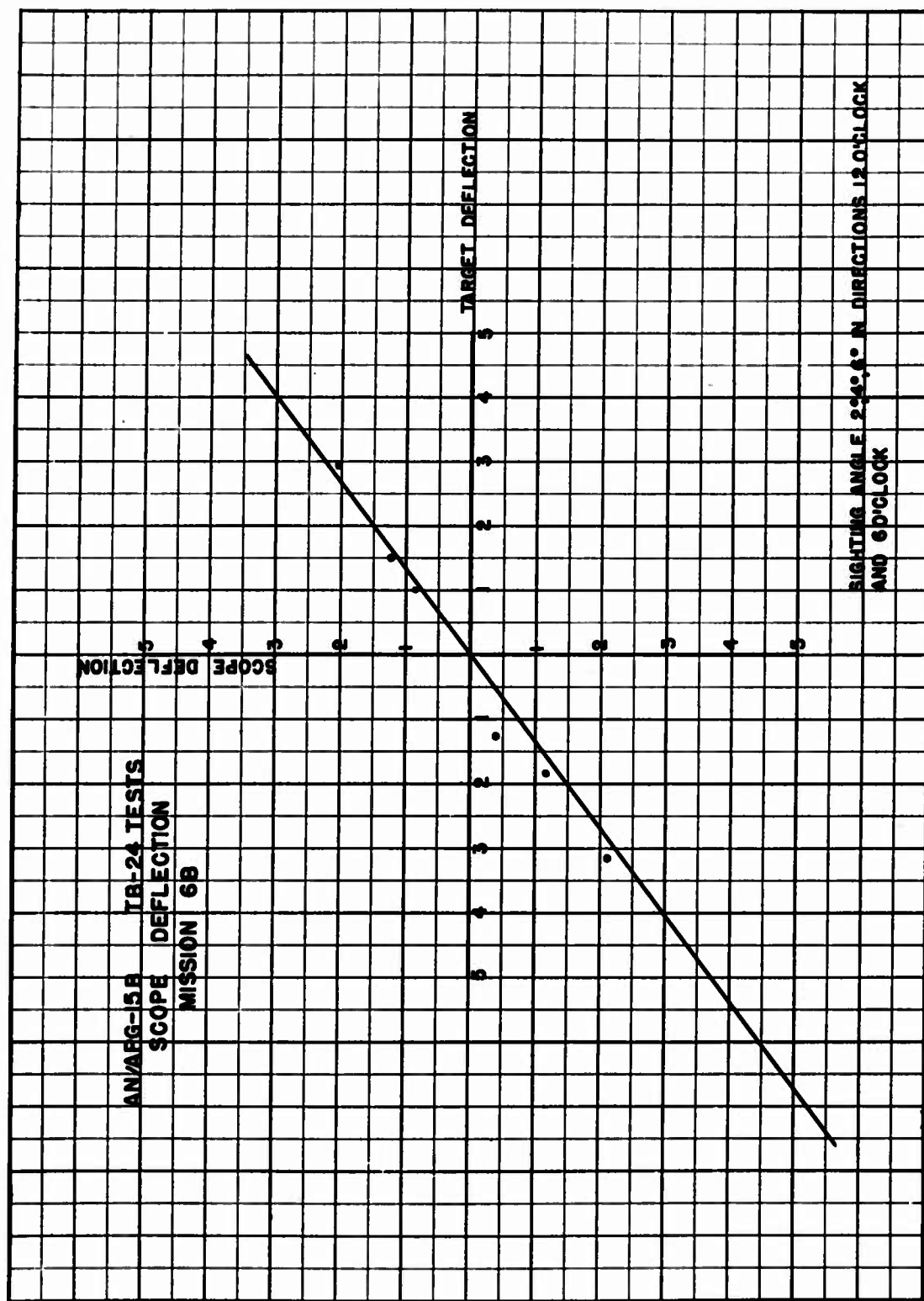
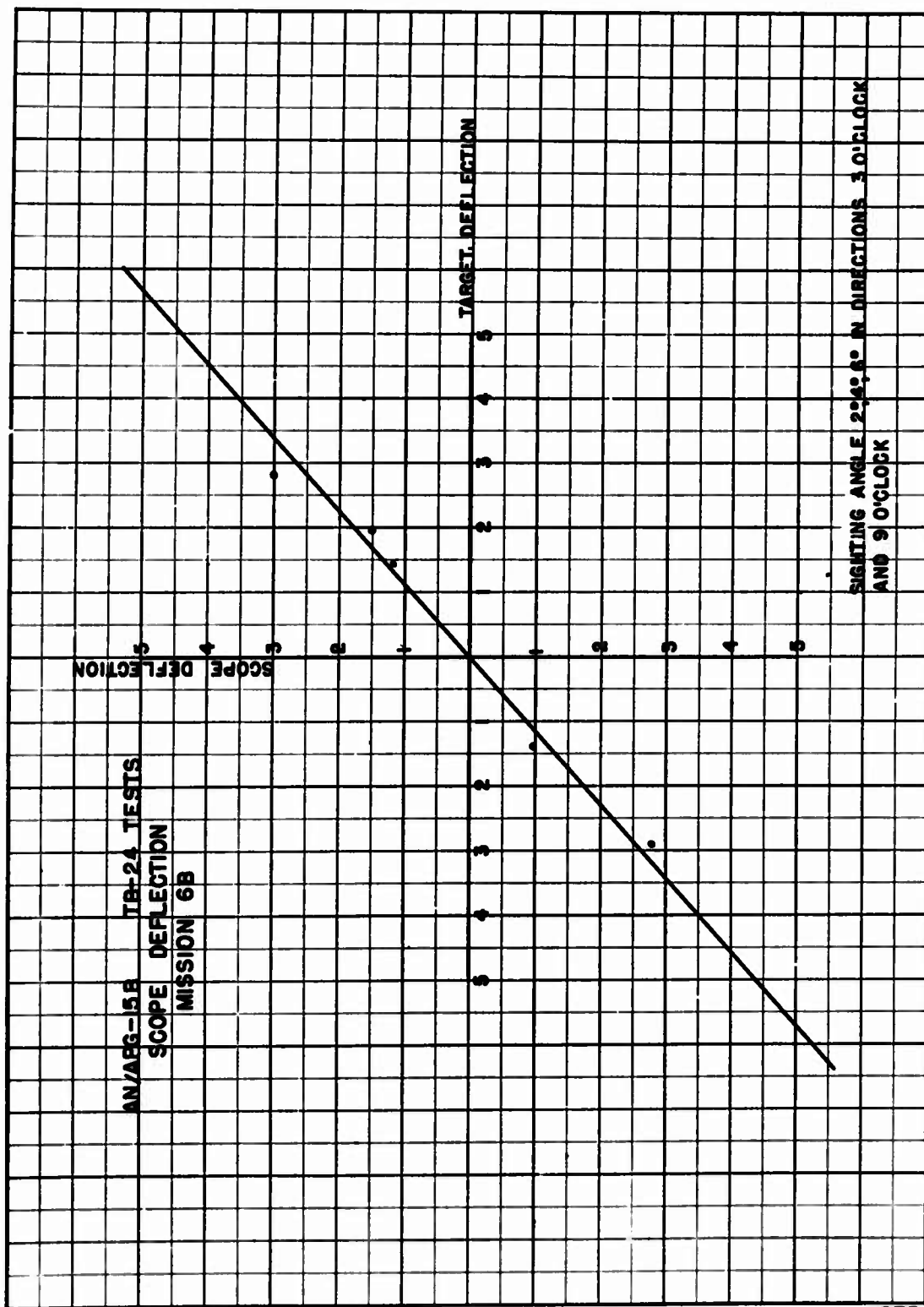


FIG. 2

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FIG. 5

Recapitulation:

Total range

<u>Target</u>		<u>Scope</u>	
Computer in	Computer out	Computer in	Computer out
mean $\sigma = 1.25$	mean $\sigma = 1.20$	mean $\sigma = .67$	mean $\sigma = .65$
$\sigma = .21$	$\sigma = .22$	$\sigma = .15$	$\sigma = .16$
c.p.e. = 1.45 in.	c.p.e. = 1.41 in.	c.p.e. = .789 in.	c.p.e. = .765 in.
n = 11	n = 10	n = 11	n = 10

Partial range, 800 to 400 yards.

<u>Target</u>		<u>Scope</u>	
Computer in	Computer out	Computer in	Computer out
mean $\sigma = 1.00$	mean $\sigma = 1.09$	mean $\sigma = .58$	mean $\sigma = .57$
$\sigma = .15$	$\sigma = .27$	$\sigma = .15$	$\sigma = .17$
c.p.e. = 1.18 in.	c.p.e. = 1.28 in.	c.p.e. = .683 in.	c.p.e. = .671 in.
n = 9	n = 9	n = 9	n = 9

The differences in standard deviation for computer in and for computer out are not significant according to the normal curve, for both target and scope. Tracking accuracy is not affected by the computer.

# VI Missions 8 and 9 : Tracking Accuracy, Computer In, Computer Out

In a series of tests designed to determine how much the computer affected the tracking (gun pointing) accuracy of AN/APG-15B, flights were made in the TB-24 bombing plane with a Navy plane F-6F, No. 298, as target. Starting at a distance of from 800 to 1000 yards from the bomber, the target plane made S-turns, crossing over the course at 30° at a range of 400 yards. With computer in, 11 runs were analyzable; with computer out, 10 runs.

The record was analyzed for the whole range, up to 1000 yards, and also for the span from 800 yards to 400 yards. Photographic range was calculated by measuring the total wing span of the image of the F-6F and by following the procedure described briefly in Missions 1, 2, 3, 4. (Actual wing span of the plane was 42.83 feet.) Coordinates of the central point of the nose of the F-6F plane (determined by inspection) were measured from the axes intersecting at the boresight point. From these measurements an ideal reference point was computed and deviations were calculated from it. Allowable percentage errors for measuring, focussing and calibrating are 2.4% at 1000 yards, 2.0% at 800 yards, 1.3% at 500 yards. Standard deviations for elevation and azimuth were found from the formula  $\sigma = 1.2533$  m.a.e. (mean absolute error) and standard deviations for target and scope were found by use of the formula

$\sigma = \sqrt{5\sigma_e^2 + \sigma_a^2}$ . The c.p.e. (circular probable error) was found

from the formula c.p.e. =  $1.1774\sigma$ . Percentage error in measurement of w, wing span, is  $\frac{dw}{w} = \frac{3}{\sqrt{10}} \frac{dR}{R}$  where R is range.

$\frac{dw}{w} = .9487 \frac{dR}{R}$  (Halmos' Error Study)

For T 25 feet

<u>R</u>	<u>% error in w</u>
500	1.33
600	1.58
700	1.90
800	2.02
900	2.21
1000	2.37

# Mission 5, Computer In. Total Range

Film Serial No.	Run	Target		Scope		$\sqrt{S}$
		Elevation m.a.e.	Azimuth m.a.e.	Elevation m.a.e.	Azimuth m.a.e.	
F-277, 291	1 171	.59	1.15	.21	.62	.58
	2 121	.57	1.34	.24	.94	.86
	3 163	.54	1.13	.21	.58	.55
	4 169	.80	1.14	.34	.66	.66
F-278, 310	1 161	.65	1.51	.33	1.01	.94
	2 185	.85	1.17	.36	.65	.65
	3 170	.40	1.05	.23	.59	.56
	4 121	.24	.84	.13	.41	.38
F-270, 167	1 239	1.02	1.08	.54	.54	.68
	2 145	.93	1.26	.46	.65	.70
	3 164	.92	1.15	.43	.78	.79
		c.p.e.=1.448 in.	Mean $\sqrt{T} = 1.23$	c.p.e.=.789 in.	Mean $\sqrt{S} = .67$	$\sqrt{S} = .15$

With an allowable error of 2% at 800 yards, 1.3% at 500 yards.

Mission 9, Computer Out. Total Range

Film Serial No.	Run N	Target		Azimuth		Elevation		Scope		Azimuth		VS
		m.s.e.	$\sqrt{e}$	m.s.e.	$\sqrt{e}$	m.s.e.	$\sqrt{e}$	m.s.e.	$\sqrt{e}$	m.s.e.	$\sqrt{e}$	
F-279, 311	1 169	.54	.75	1.63	2.04	1.52	1.27	.31	.95	1.07	.73	
	2 175	.60	.75	.91	1.14	.96	.22	.23	.44	.55	.44	
	3 127	.58	.73	1.20	1.50	1.13	.27	.34	.59	.74	.53	
	4 134	.47	.57	.97	1.22	.95	.24	.30	.49	.61	.48	
F-283, 297	1 164	.37	1.09	.67	.84	.97	.29	.27	.37	.46	.41	
	2 160	.40	.50	1.02	1.35	1.01	.24	.30	.60	.85	.64	
	3 107	.68	.85	1.17	1.47	1.20	.24	.30	.95	1.19	.87	
F-274, 285	1 214	.63	.79	1.34	1.68	1.34	.32	.40	.74	.93	.71	
	2 238	.91	1.14	1.23	1.54	1.35	.46	.58	.77	.83	.72	
	3 185	1.12	1.40	1.40	1.75	1.58	.60	.75	.74	.93	.74	

c.p.e. = 1.415 in.

Mean  $\sqrt{e}$  = 1.20

c.p.e. = .775 in. Mean  $\sqrt{e}$  = .55

Mission 2, Computer In. 400-800 Yards

Film Serial No.	Run	Target		Scope		Azimuth	Elevation	m.a.e.	σ <sub>s</sub>	σ <sub>s</sub>	σ <sub>s</sub>
		Flevation	σ <sub>s</sub>	m.a.e.	σ <sub>s</sub>						
F-277, 291	1	105	.80	.57	1.22	1.02	.23	.29	.53	.66	.51
	2	77	.59	1.20	1.50	1.14	.26	.33	.92	1.15	.84
	3	77	.60	1.05	1.32	1.02	.19	.24	.60	.75	.56
	4	112	.91	.79	.99	.95	.28	.34	.43	.54	.45
F-278, 310	1	81	.45	1.24	1.58	1.16	.31	.39	.52	1.03	.78
	2	103	.88	.91	1.14	1.02	.32	.41	.57	.71	.59
	3	98	.39	.72	.90	.69	.15	.19	.43	.54	.40
	4	94	.25	.85	1.06	.77	.13	.14	.42	.53	.39
F-270, 147	1	82	.79	1.02	1.35	1.18	.61	.72	.54	.68	.72
		c.p.e. = 1.177 in.	Mean	1.00	σ <sub>s</sub> = 1.15	c.p.e. = 2.685 in.	Mean	15.50	σ <sub>s</sub> = 15		

With an allowable error due to measuring, focussing, calibrating of 2.5 at 800 yards, 1.3% at 500 yards.

Mission 9, Computer Out. 200-400 Yards.

Film Serial No.	Run N	Target		Scope		Azimuth	VS
		Elevation m.a.e.	Azimuth m.a.e.	Elevation m.a.e.	Azimuth m.a.e.		
F-279, 211	1 90	.40	.50	.22	.28	.90	.67
	2 87	.47	.59	.17	.21	.44	.34
	3 73	.40	.50	.21	.36	.56	.44
	4 90	.46	.58	.24	.30	.51	.42
F-283, 297	1 95	.87	1.09	.35	.44	.35	.35
	2 87	.39	.49	.72	.26	.75	.55
	3	Information to 602 yards only					
F-274, 285	1 42	.57	.71	.40	.50	.67	.69
	2 99	.94	1.16	.50	.63	.58	.58
	3 71	1.22	1.53	.74	.93	.73	.92
n 9		c.p.e. = 1.283 in.	Mean $\sqrt{\frac{VS}{n}} = 1.09$	c.p.e. = .671 in.	Mean $\sqrt{\frac{VS}{n}} = .57$		
			$\sqrt{\frac{VS}{n}} = .27$				

# Appendix Note on Time Constant

If a system is described by a linear differential equation of the form (1)  $K \frac{d\lambda}{dt} + \lambda = f(t)$ , where  $\lambda = \lambda(t)$  is the amplitude of the motion of the pip on the scope,  $f(t)$  is a function appropriate to the system, and  $K$  is a constant called the time constant, a physical meaning is readily ascribed to  $K$ .  $f(t)$  might well be the  $\frac{d\mu}{dt}$  where  $\mu$  is the amplitude of the motion of the image of the target. Suppose  $f(t) = 0$ . The equation

(2)  $K \frac{d\lambda}{dt} + \lambda = 0$  yields the general solution  $\lambda = \lambda_0 e^{-t/K}$ , where  $\lambda_0$  is the amplitude on the scope for  $t = 0$ . When  $t = K$ ,  $\lambda = \frac{\lambda_0}{e}$ . That is, the time constant  $K$  is the value of  $t$  for which the amplitude is the original amplitude divided by  $e$ .

The graphs of readings for Missions 7 and 12 show that the motion is roughly sinusoidal. Assuming that the motion is sinusoidal, let  $f(t) = \sin \omega t$  where  $\omega$  is  $2\pi f$ , where  $f$  is the frequency. The differential equation takes the form

$$(3) \quad K \frac{d\lambda}{dt} + \lambda = \sin \omega t$$

The general solution of (3) is the sum of the general solution of the homogeneous equation (2) and a particular solution of (3). Let

$\lambda = B \sin(\omega t - \phi)$  where  $B$  and  $\phi$  are constants to be determined.

$$\text{Then } K \frac{d\lambda}{dt} + \lambda = B [k\omega \cos(\omega t - \phi) + \sin(\omega t - \phi)] = \sin \omega t$$

$$= B [k\omega \cos \phi - \sin \phi] \cos \omega t + B [k\omega \sin \phi + \cos \phi] \sin \omega t$$

$$(4) \quad B [k\omega \cos \phi - \sin \phi] = 0$$

$$(5) \quad B [k\omega \sin \phi + \cos \phi] = 1$$

$$\text{Equation (4) yields } \tan \phi = k\omega \text{ and Equation (5) yields } B = \frac{1}{\sqrt{1 + k^2 \omega^2}}$$

The general solution of equation (3) is

$$\lambda = \frac{1}{\sqrt{1 + k^2 \omega^2}} \sin [\omega t - \tan^{-1} k\omega] + C e^{-t/K},$$

where  $C$  is an arbitrary constant.

In Missions 7 and 12, the time lag measured is  $K' = \phi/\omega$  as is easily seen from the function  $\sin\omega(t - \phi/\omega)$  in which the shift in time is  $\phi/\omega$ . Then  $\phi = K'\omega$  and  $K = \frac{\tan(K'\omega)}{\omega}$ .

This formula establishes a relationship between  $K$ , the time constant of the system, and  $K'$ , the time lag, for the linear, sinusoidal system.

Now,  $\tan(K'\omega) = (K'\omega) + \frac{(K'\omega)^3}{3} + \frac{2(K'\omega)^5}{15} + \dots$  which

converges for  $-\pi/2 < K'\omega < \pi/2$ .

$$K = K' + \frac{K'^3 \omega^2}{3} + \frac{2K'^5 \omega^4}{15} + \dots \quad -\pi/2 < K'\omega < \pi/2$$

It is clear that  $K > K'$  and for small values of  $K'\omega$   $K$  is nearly equal to  $K'$ .

G. T. Bumer  
November 30, 1945

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Flights to test the range and tracking accuracy of AN/APG-15B radar airborne detector were made in a TB-24 bombing plane with various planes as targets. The film record of readings of radar range dials and oscilloscopes was compared with the photographs of the target planes taken from the tail of the bomber. Analysis shows that the AN/APG-15B measures air-to-air range as well as photographic sighting. It was established that the tracking or gun pointing accuracy is unaffected by the computer. An approximate linear relation is indicated between scope and target deflection.

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